Philosophy 211 -- Assignment #5
Supplement

Assignment 5 deals with Truth Tables and Truth-Value Assignments. This is our only homework that deals with Chapter 2 of *Logic Primer*.

A Truth-Value Assignment (TVA) is a function that assigns either T or F to each of the sentence letters in the language. A Truth Table is a way of systematically representing all possible truth-value assignments.

For example, even though there are an infinite number of TVAs, half of these TVAs assign T to P and half assign F to P. If P is the only sentence letter that we are interested in, we can divide all of the TVAs into two classes: ones that assign T to P and ones that assign F to P. If there are two letters that we are concerned with, say P and Q, there are four relevant classes of TVAs: ones that assign T to both P and Q, ones that assign T to P and F to Q, ones that assign F to P and T to Q, and ones that assign F to both P and Q.

Since each of the connectives in our language is truth-functional, the truth-value that a TVA assigns to a sentence is completely determined by the truth-values of its parts. For example, if P is T and Q is F, then the sentence Pv(~Q→P) is true since P is true and a disjunction is true when at least one of its disjuncts is true – (as it happens, the other disjunct is also true.)

To determine the truth-values of complex sentences, we need to know how each of our connectives takes combinations of truth-values and turns them into a single value. Here are the characteristic truth-tables for each of our connectives:

P	~P
T	F
F	T

P, Q	PvQ
T T	T
T F	T
F T	T
F F	F

P, Q	P&Q
T T	T
T F	F
F T	F
F F	F

P, Q	P→Q
T T	T
T F	F
F T	T
F F	T

P, Q	P↔Q
T T	T
T F	F
F T	F
F F	T

We can produce a truth-table for a sentence by systematically figuring out the value of each part of the sentence. For example, lets produce a truth-table for the sentence (P→Q)&~P.

First, I would write the outline of the table – the sentence on top and each of the four possible TVAs on the left side.

P, Q	(P→Q)& ~P
T T	
T F	
F T	
F F	

Now, write down the truth-values of the parts of the sentences (inside then out) as determined by the truth-values of P and Q given on the left of each row. Lets start with the first conjunct P→Q:

P, Q	(P→Q)& ~P
T T	T
T F	F
F T	T
F F	T

The entire sentence is a conjunction of two smaller sentences so I need to figure out the value of each conjunct before I can determine the value of the entire conjunction. So let's figure out the value of ~P on each of these rows:

P, Q	(P→Q)	& ~P
T T	T	F
T F	F	F
F T	T	T
F F	T	T

Now to determine the value of the entire conjunction, I simply calculate the value of a conjunction with T, F on the first row; F,F on the second row, etc.

P, Q	(P→Q)	&	~P
T T	T	F	F
T F	F	F	F
F T	T	T	T
F F	T	T	T

I have now determined that when P and Q are both true, (P→Q)&~P is false (first row.) When P is T and Q is F, (P→Q)&~P is false (second row.) etc.

To save time, when testing multiple sentences, simply put them on the same table:

P, Q	(P→Q)&		~P	PvQ
T T	T	F	F	T
T F	F	F	F	T
F T	T	T	T	T
F F	T	T	T	F

Now, there are several questions that you can answer by looking at truth-tables. Among them are, is a given sequent valid, is a given set of sentences consistent, are two given sentences equivalent, and is this one sentence a tautology.

Here are the relevant definitions to use with the tables:

A sequent is <u>invalid</u> if there is a TVA that makes each of its premises true and its conclusion false. A sequent is <u>valid</u> if there is no such TVA.

A set of sentences is <u>consistent</u> if there is at least one TVA that makes each of the sentences in that set true.

Two sentences are truth-functionally <u>equivalent</u> if they have the same truth-value on every TVA.

A sentence is a <u>tautology</u> if it is true on every TVA.
A sentence is <u>inconsistent</u> if it is false on every TVA.
A sentence is <u>contingent</u> if it is neither tautologous nor inconsistent.

EXAMPLE 1

P, Q	(P→Q)	&	~P	PvQ	Q
T T	T	F	F	T	T
T F	F	F	F	T	F
F T	T	T	T	T	T
F F	T	T	T	F	F

This table shows that the sequent (P→Q)&~P, PvQ ├ Q is a valid sequent since there is no row of the truth-table where both of the premises are true and the conclusion is false. However, (P→Q)&~P, PvQ ├ P is not a valid sequent. Both premises are true and the conclusion is false on the F, T row (third row) of the table (See example 2).

EXAMPLE 2

P, Q	(P→Q)&~P	PvQ	P
T T	T F F	T	T
T F	F F F	T	T
F T	T T T	T	F
F F	T T T	F	F

Example 1 shows that the set (P→Q)&~P, PvQ, Q is a consistent set since each member is true on the third row of the table. However, (P→Q)&~P, PvQ, P is not a consistent set since there is no row on the table where each member is true. This is due to the fact that the first sentence forces ~P to be true and thus P to be false whereas the last member is simply P which is false when ~P is true.

Two sentences are equivalent when they share the same table. By 'same table' I simply mean the column under the main connective is the same.

EXAMPLE 3:

PvQ is equivalent to ~P→Q since they share the same table.

P, Q	PvQ	~P→ Q
T T	T	F T T
T F	T	F T F
F T	T	T T T
F F	F	T F F

EXAMPLE 4:

~PvP is a tautology, P&~P is contradictory, and P is contingent:

P, Q	~P v P	P & ~P	P
T T	F T T	T F F	T
T F	F T T	T F F	T
F T	T T F	F F T	F
F F	T T F	F F T	F